Numeric Response Questions

Matrices

Q.1 If three matrices $A=\begin{bmatrix}2 & 1\\ 4 & 1\end{bmatrix}, B=\begin{bmatrix}3 & 4\\ 2 & 3\end{bmatrix}$ and $C=\begin{bmatrix}3 & -4\\ -2 & 3\end{bmatrix}$.

Then $t_r(A)+t_r\left(\frac{ABC}{2}\right)+t_r\left(\frac{A(BC)^2}{4}\right)+t_r\left(\frac{A(BC)^3}{8}\right)+\cdots\infty=$

Q.2 If A is a square matrix of order 3 such that $A(\text{adj. }A)=\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}$, then find $|\text{adj. }A|$.

Q.3 If $A^2=A$ and $(I+A)^4$ is equal to $I+kA$ then find k.

Q.4 If $A=\begin{bmatrix}1 & -1 & 1\\ 0 & 2 & -3\\ 2 & 1 & 0\end{bmatrix}, B=(\text{adj }A)$ and $C=5A$, then find $\frac{|\text{adj }B|}{|C|}$.

Q.5 If $U=[2,-3,4], V=\begin{bmatrix}3\\ 2\\ 1\end{bmatrix}, X=[0,2,3], Y=\begin{bmatrix}2\\ 2\\ 4\end{bmatrix}$ and $UV+XY=[k]$ then find k.

Q.6 If A is a square matrix of order n such that $|\text{adj }(\text{adj }A)|=|A|^9$, then find the value of n.

Q.7 If $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}, A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}, Q=PAP^T$ and $P^QQ\times P=\begin{bmatrix}1 & k\\ 0 & 1\end{bmatrix}$ then find k.

Q.8 If $A=\begin{bmatrix}1 & 2 & 3\\ -2 & 3 & -1\\ 3 & 1 & 2\end{bmatrix}$ and I is a unit matrix of 3^{rd} order then find the element azs of (A^2+9I).

Q.9 In a symmetric matrix of order ' 12 ', find maximum number of different elements.

Q.10 A is a 2×2 matrix such that $A\begin{bmatrix}1\\ -1\end{bmatrix}=\begin{bmatrix}-1\\ 2\end{bmatrix}$ and $A^2\begin{bmatrix}1\\ -1\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$. Find the sum of the elements of A.

Q.11 Let $A=\begin{bmatrix}a & b & c\\ b & c & a\\ c & a & b\end{bmatrix}$ and $|A|=1, abc=1$, then find the value of $a^5+b^3+c^3$

Q.12 Find the sum of the values of x satisfying the equation

$$[1\quad 105]\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}^4\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}^2\begin{bmatrix}x^2-3x+1\\ 2\end{bmatrix}=[50].$$

Q.13 If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and A^{100} is equal to $2^k A$ then find k.

Q.14 If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ and det [adj (adj A)] is equal to k^2 then find $(k + \lambda)$.

Q.15 The number of different matrices which can be formed using 12 different real numbers is m(12)! then find the value of m.

ANSWER KEY

1. 6.00	**2.** 4.00	**3.** 15.00	**4.** 1.00	**5.** 20.00	**6.** 4.00	**7.** 2005.00
8. 13.00	**9.** 78.00	**10.** 5.00	**11.** 2.00	**12.** 3.00	**13.** 99.00	**14.** 18.00
15. 6.00						

Hints & Solutions

1. $BC = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$$t_r(A) + t_r\left(\frac{ABC}{2}\right) + t_r\left(\frac{A(BC)^2}{4}\right) + t_r\left(\frac{A(BC)^3}{8}\right) + \ldots\ldots + \infty$$

$$= t_r(A) + t_r\left(\frac{A}{2}\right) + t_r\left(\frac{A}{4}\right) + \ldots\ldots \infty$$

$$= t_r(A) + \frac{1}{2}t_r(A) + \frac{1}{2^2}t_r(A) + \ldots\ldots \infty$$

$$= \frac{t_r(A)}{1-1/2} = 2t_r(A) = 2(3) = 6$$

2. $|A \,.\, \text{adj } A| = 8$

$|A|\ |\text{adj } A| = 8$

$\Rightarrow\ |A|^3 = 8$

$\Rightarrow\ |A| = 2 \ \Rightarrow\ |\text{adj } A| = 4$

3. $(I + A)^4 = (I + A)^2 . (I + A)^2$

$= (I + 2A + A^2) . (I + 2A + A^2)$

$= (I + 3A) . (I + 3A)$

$= I + 6A + 9A^2$

$= I + 6A + 9A$

$= I + 15A$

4. $|A| = 5$

Now $|\text{adj } B| = |\text{adj.(adj}A)|$

$= |A|^4 = 5^4$

$|C| = |5A| = 5^3|A| = 5^4$

$\therefore \dfrac{|\text{adj. } B|}{|C|} = 1$

5. $[2\ \ -3\ \ 4]\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + [0\ \ 2\ \ 3]\begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$

$= [6 - 6 + 4] + [0 + 4 + 12] = [4] + [16]$

$= [20]$

6. $|\text{Adj Adj } A| = |A|^{(n-1)^2} = |A|^9$

$(n - 1) = 3$ or -3

$n = 4$ or -2

$n = 4$ only

7. Given that

$P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix},\ A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$Q = PAP^T$ and $X = P^TQ^{2005}P$

We have,

$P^TP = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

We observe that

$Q = PAP^T$

$\Rightarrow Q^2 = (PAP^T)(PAP^T)$

$= PA(P^TP)AP^T = PA(IA)P^T$

$= PA^2P^T$

Proceeding in the same way, we get

$Q^{2005} = PA^{2005}P^T$

Also, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

And proceeding in the same way,

$A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

Now, $X = P^TQ^{2005}P$

$= P^T(PA^{2005}P^T)P = (P^TP)A^{2005}(P^TP)$

$= IA^{2005}I = A^{2005}$

$= \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

8. $A^2 = \begin{bmatrix} 6 & 11 & 7 \\ -11 & 4 & -11 \\ 7 & 11 & 12 \end{bmatrix}$

$A^2 + 9I = \begin{bmatrix} 15 & 11 & 7 \\ -11 & 13 & -11 \\ 7 & 11 & 21 \end{bmatrix}$

9. Maximum no. of different elements

$= \frac{n^2+n}{2} = \frac{144+12}{2} = \frac{156}{2} = 78$

10. $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ ...(1)

$A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ...(2)

Let A be given by $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

The first equation gives

$a - b = -1$ ...(3)

and$c - d = 2$ ...(4)

For second equation, $A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= A\left(A\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$

$= A\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

This gives $-a + 2b = 1$ ....(5)

and $-c + 2d = 0$ ...(6)

(3) + (5) $\Rightarrow b = 0$ and $a = -1$

(4) + (6) $\Rightarrow d = 2$ and $c = 4$

so the sum $a + b + c + d = 5$

11. $3abc - (a^3 + b^3 + c^3) = 1$

$\Rightarrow a^3 + b^3 + c^3 = 2$

12. $[1 \quad 105]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x^2-3x+1 \\ 2 \end{bmatrix} = [50]$

$\Rightarrow [1 \quad 105]\begin{bmatrix} x^2-3x+1 \\ 2 \end{bmatrix} = [50]$

$\Rightarrow x^2 - 3x + 1 + 210 = 50$

$\Rightarrow x^2 - 3x + 161 = 0$

$\therefore$ sum of the roots = 3

13. $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A^2 = 2.A \Rightarrow A^{1.0} = 2^{99} A$

$A^3 = 2^2 A$

14. $|A| = 14$

$\therefore |adj. (adj A)| = 14^4$

15. Possible orders are

$12 \times 1, 1 \times 12, 6 \times 2, 2 \times 6, 4 \times 3, 3 \times 4$

So total no. of different matrices = $6\lfloor 12$